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An experimental investigation of the turbulent mixing transition in the Richtmyer-Meshkov instability

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The Richtmyer-Meshkov instability (RMI) is experimentally investigated in a vertical shock tube using a broadband initial condition imposed on an interface between a helium-acetone mixture and argon ($A \approx 0.7$). The interface is created without the use of a membrane by first setting up a flat, gravitationally-stable stagnation plane, where the gases are injected from the ends of the shock tube and exit through horizontal slots at the interface location. Following this, the interface is perturbed by injecting gas within the plane of the interface. Perturbations form in the lower portion of this layer due to the shear between this injected-stream and the surrounding gas. This shear layer serves as a statistically-repeatable broadband initial condition to the RMI. The interface is accelerated by either a $M = 1.6$ or $M = 2.2$ planar shock wave, and the development of the ensuing mixing layer is investigated using planar laser-induced fluorescence (PLIF). The PLIF images are processed to reveal the light-gas mole fraction by accounting for laser absorption and laser-steering effects. The images suggest a transition to turbulent mixing occurring during the experiment. An analysis of the mole-fraction distribution confirms this transition, showing the gases begin to homogenize at later times. The scalar variance energy spectrum exhibits a $k^{-5/3}$ inertial range, providing further evidence for turbulent mixing. Measurements of the Batchelor and Taylor microscales are made from the mole-fraction images, giving ∼150 μm and 4 mm, respectively, by the latest times. The ratio of these scales implies an outer-scale Reynolds number of 6-7×10^4.

1. Introduction

A shock wave passing through an interface between two densities will deposit vorticity on interfacial perturbations. This interaction, known as the Richtmyer-Meshkov instability (RMI) (Richtmyer 1960; Meshkov 1970), leads to the unbounded growth of perturbations and can result in turbulent mixing. The RMI is akin to the Rayleigh-Taylor instability (RTI) (Rayleigh 1883; Taylor 1950), where a finite acceleration causes an interface to become unstable. These instabilities can lead to mixing of material interfaces in inertial confinement fusion capsules (Lindl *et al.* 2004), the formation of supernova remnants (Kane *et al.* 1999), and enhanced fuel-oxidizer mixing in supersonic combustion (Marble *et al.* 1987).

The RMI starts with baroclinic production of vorticity, where a misalignment of the pressure gradient, ∇p, at the shock front and the density gradient, $\nabla \rho$, at the interface will lead to vorticity (ω) production; $D\omega/Dt = (\nabla\rho \times \nabla p)/\rho^2$. The perturbations grow linearly until their amplitudes become comparable to their wavelengths. The interface can develop into a turbulent mixing layer if the initial perturbation contains a broad range of scales and it is accelerated by a sufficiently strong shock.

Turbulent mixing requires a separation between the largest, energy-containing scales and the smallest, dissipative scales. The controlling length scales are the Liepmann-Taylor scale λ_L (related to the Taylor microscale) and the inner viscous scale λ_ν (related to the Kolmogorov scale) (Dimotakis 2000). The Liepmann-Taylor scale is effectively the smallest scale generated by the largest eddies, while the inner viscous scale is the scale where energy begins to be removed through viscous dissipation. Thus when $\lambda_L > \lambda_\nu$, turbulent mixing is expected. In steady-state flows, these scales are related to the Reynolds number through

$$\lambda_L = 5\mathcal{L}Re^{-1/2} \tag{1.1}$$

$$\lambda_\nu = 50\mathcal{L}Re^{-3/4}\,, \tag{1.2}$$

where $\mathcal{L}$ is the largest scale of the flow, which implies a transition Reynolds number of $Re = 1\text{-}2 \times 10^4$.

For time-dependent flows like the RMI, the Reynolds number can exceed the turbulent transition before the flow develops the scale separation necessary for turbulent mixing. Robey *et al.* (2003) considered the case where the interface starts as a discontinuity and therefore the Taylor microscale begins at zero. With the Taylor microscale growing with time as $\lambda_T \propto \sqrt{\nu t}$, eventually it surpasses the viscous scales, marking a transition to turbulence. In the experiments discussed here, a different scenario occurs: the Taylor and viscous scales start at finite values set by the initial condition. After the shock interaction, where additional energy is deposited in the layer, these scales evolve towards their fully-developed values.

Previous works have observed evidence of turbulent mixing in shock tube experiments. In a shock-accelerated gas curtain, Rightley *et al.* (1999) identified a turbulent transition when the intensity histogram of planar post-shock images no longer showed a local peak of unmixed fluid. Vorobieff *et al.* (1998, 2003) used second-order structure functions on images from similar experiments to identify turbulent mixing. The power-law slope of the structure function at late times was analogous to a $k^{-5/3}$ spectrum in wavenumber space. Recent work by Balakumar *et al.* (2012) used simultaneous density and velocity measurements to study the turbulent behavior after a second shock wave interacted with the layer. For single-interface RMI, Zhou *et al.* (2003) found that the single-mode experiments of Jones & Jacobs (1997) and Collins & Jacobs (2002) approached the transition limit at the latest times of their highest Mach number experiment, where the vortex cores exhibited a chaotic structure. In an earlier subset of the present work (Weber *et al.* 2012), the shock-induced mixing layer appeared to be transitioning to turbulent mixing at the latest observed time. For the present experiments, the shock tube test section was extended to observe the mixing layer after this transition. Additionally, a higher Mach number campaign was undertaken for a point of comparison.

In the experiments presented here, evidence for a transition to turbulent mixing of the shock-accelerated mixing layer is inferred by a homogenization of the mole-fraction probability density functions (PDFs) and the emergence of an inertial range in the scalar energy spectra. The length scales are measured and found to separate with time. The largest scale, the overall thickness of the mixing layer, initially grows linearly, the smallest scale, the Batchelor scale, decreases in size, and the intermediate Taylor microscale remains nearly constant. Coincident with the inferred turbulent transition, the Taylor microscale and the Batchleor scale appear sufficiently separated to sustain an inertial range. The paper begins with a description of the experimental setup, the initial condition, and the image processing. The results section describes the structure of the mixing layer as it evolves in time. Length scales are extracted from the mixing layer using several methods and are finally used to estimate the Reynolds number.

2. Experimental Setup

The present experiments were performed at the Wisconsin Shock Tube Laboratory. The shock tube is 9.1 m tall and has a 25.4×25.4 cm^2 internal cross section. The 2.0 m driver section is separated from the rest of the shock tube by a steel diaphragm. Before each experiment, the driver section is pressurized to 85% of the diaphragm rupture pressure. The remaining pressure is rapidly provided through two pneumatically driven, fast-opening valves. The rupture of the diaphragm releases a shock wave into the atmospheric pressure gas below. The distance between the diaphragm and the interface, 5.4 m, allows the shock wave to stabilize and become planar before interacting with the interface. The bottom section of the shock tube contains ports for generating the interface and windows for planar imaging. Four window ports are used in the present work. The top-most window is positioned to view the initial condition and an early post-shock time. Lower windows allow for later-time visualization. These windows are made of fused silica and are 7.5 cm thick to withstand the dynamic loading by strong shock waves. The end wall of the shock tube contains a rectangular window to transmit the laser sheets used for flow visualization.

These experiments use a gas interface with a mixture of helium and acetone vapor (6.0 ± 0.8% by volume) above and pure argon below, giving an Atwood number of $A = (\rho_2 - \rho_1)/(\rho_2 + \rho_1) = 0.7$. The flow of the helium-acetone mixture is split, routing a portion to the top of the shock tube and the remaining to the interface section. First, an initially flat interface is formed by flowing the helium-acetone mixture into the top of the shock tube and argon into the bottom. Excess gas is evacuated through slots in the shock tube wall at the interface location. These slots are connected to a pair of vacuum pumps, ensuring a rapid outflow of gas. This method to create a flat, membrane-less interface is similar to that developed for the University of Arizona shock tube (Jones & Jacobs 1997) and used previously at the University of Wisconsin (Motl *et al.* 2009).

The flat interface is perturbed by injecting the pure argon and the helium-acetone mixture horizontally through separate slots above and below the stagnation plane, respectively, while maintaining the vertical flow started previously. This flow configuration, shown in Fig. 1, was experimentally determined to provide the best initial condition in terms of scale content and statistical repeatability. Perturbations form due to the buoyant interaction between the two streams and from the shear stress between this mixed layer and the pure argon. The superposition of the horizontal and vertical flows creates a continual flow towards the interface, ensuring that all mixed gas is removed and the mixing layer remains statistically steady in time.

Two excimer lasers (Lambda Physik LPX 210i, 308 nm, 470 mJ/pulse, 28 ns pulse) are used for planar laser-induced fluorescence (PLIF) diagnostics. During each experiment, ten pre-shock images are recorded prior to the arrival of the shock wave to obtain a statistical description of the initial condition. To allow the laser to recharge and account for variability in experimental timing, the last recorded initial condition occurs 150-200 ms prior to the shock arriving at the interface. A pressure transducer above the interface is used to trigger the two lasers for two post-shock images based on the arrival of the shock wave. The images are all recorded using three thermoelectrically cooled (to -60°C) Andor CCD cameras (model DV434-BU2).

Figure 2(a) shows a sample initial condition image, corrected so that the signal intensity

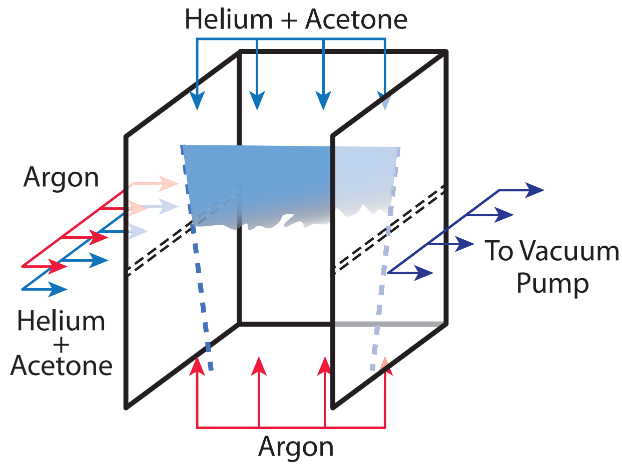

FIGURE 1. Diagram of the interface location showing gas flowing from the top and bottom of the shock tube and from the left set of slots. The right set of slots are connected to a vacuum pump, removing excess gas. The edges of the planar laser sheet are shown as dashed lines and the laser causes fluorescence in the acetone vapor in the top gas.

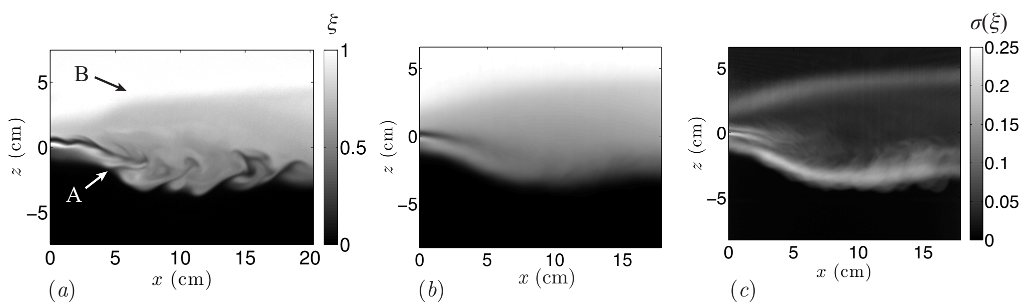

FIGURE 2. Initial condition images. (*a*) A sample initial condition image, processed so intensity corresponds to light-gas mole fraction. (*b*) Mole fraction ensemble average and (*c*) standard deviation, σ, from 100 images

corresponds to acetone concentration, which is also directly proportional to the light-gas mole fraction, ξ. In the image, the gases are injected from the left near $z = 0$ cm. The injected stream of pure argon is visible as a dark horizontal band at $z \approx 0.5$ cm. The injected helium-acetone mixture is visible below the argon stream. Approximately 5 cm to the right right of the injection location (feature A), the two gas streams begin mixing and the individual streams are no longer apparent. Perturbations develop on the lower edge of this mixing region due to the velocity difference of the mixture stream and the ambient argon. The boundary between the mixed gas entering from the left and the helium-acetone mixture entering from the top of the shock tube is identified as feature B. The gradient at the top of the mixing region is diffuse and lacks noticeable perturbations. Between the top contour (feature B) and the bottom shear surface (feature A), the average mole fraction is $\xi \approx 0.6$. The ensemble average and standard deviation from 100 images are shown in Figs. 2(*b*) and (*c*). The most significant temporal fluctuations occur at the bottom shear surface, where the light-gas mole fraction has a standard deviation of 0.2. This initial condition is characterized further in Weber *et al.* (2012).

The interface is accelerated by an incident shock wave of strength $M = 1.57 \pm 0.02$ or $M = 2.23 \pm 0.02$. At each Mach number, images from four post-shock times are obtained (termed: PS1-PS4). These post-shock times correspond to different shock-tube window locations. Two post-shock images are obtained per experiment and a total of 20-40 images were collected at each time over the course of the experimental campaign.

The PLIF images are processed to extract the light-gas mole fraction, ξ. The images are processed using the knowledge that the top portion of the image contains pure seeded (light) gas ($\xi = 1$). Integrating downward while accounting for the divergence of the laser sheet, deviations from Beer's law attenuation are attributed to mixing of unseeded (heavy) gas or changes in temperature. The equation for this is

$$\xi = \frac{\dfrac{T}{T_1} S_f}{S_{f,R} - n_1 \sigma \phi \displaystyle\int_r^R \frac{S_f}{\phi} dr}, \tag{2.1}$$

where S_f is the local fluorescence signal, $S_{f,R}$ is the fluorescence signal at the top of the image where it is assumed $\xi = 1$, T/T_1 is the temperature ratio in relation to the pure seeded region, $n_1\sigma$ is the product of number density and absorption cross section in the pure seeded gas (this product is measured by the exponential signal variation in the top of the image), and ϕ is the fluorescence quantum yield. The integral is carried out from the local location r to the top of the image at location R. This process (without the inclusion of temperature effects) is similar to that used by Collins & Jacobs (2002) and Motl *et al.* (2009). Since the temperature is not known, it is assumed proportional to the mole fraction,

$$T = T_2' + (T_1' - T_2')\xi, \tag{2.2}$$

where T_1' and T_2' are the post-shock temperatures in the pure light and heavy gases, respectively, calculated from 1D gas dynamics. With a Prandtl number near unity, this approximation is expected to be accurate to first order, but it neglects higher order effects such as shock focusing, which can lead to higher temperatures in localized regions. Since the right side of Eq. (2.1) contains ξ in the relation for T, $\sigma(T)$, and $\phi(T)$, this equation is iteratively solved until a converged mole fraction field is found. Once the images have been corrected, fine scale features remain due to refraction of light rays caused by index of refraction gradients in the mixing layer. These features are removed through a 2D spectral notch filter. This filtering process preserves the original spectrum when applied to synthetic turbulent images (Weber 2012).

Some of the relevant gas properties are reported in Table 1. These are computed based on the measured incident and transmitted wave speeds (W_i and W_t). Primes denote post-shock quantities.

3. Experimental Results and Discussion

3.1. *Structure of the Post-Shock Scalar Fields*

Figures 3 and 4 show a sample of corrected PLIF images from the $M = 1.6$ and $M = 2.2$ experiments, respectively. The laser sheet in the late-time location, near the end wall of the shock tube, is narrower than locations further from the end wall. Because of this, all images in these figures are cropped to the PS4 width (14 cm). The first rows in each figure show three initial condition images. In Fig. 3 the second, third, fourth, and fifth rows of images are at post-shock times of 0.14 ms, 0.88 ms, 2.16 ms, and 3.84 ms, respectively. The post-shock times in Fig. 4 are 0.10 ms, 0.44 ms, 1.12 ms, and 2.05 ms. Images in the same column in rows four and five are from the same experiment, whereas all other images are from different experiments.

TABLE 1. Gas properties for the two Mach number cases. Primes denote post-shock quantities. Gas 1 is the light gas (helium seeded with acetone) and gas 2 is the heavy gas (argon). V_0 is the post-shock interface velocity.

M_i	1.57	2.23
M_t	1.85	2.88
W_i (m/s)	1150	1576
W_t (m/s)	592	919
V_0 (m/s)	315	606
Acetone (% Vol.)	5.6	6.5
ρ_1 (kg/m^3)	0.29	0.31
ρ_2 (kg/m^3)	1.63	1.63
ρ_1' (kg/m^3)	0.69	1.19
ρ_2' (kg/m^3)	3.48	4.79
T_1' (K)	497	761
T_2' (K)	557	1011
$p_1' = p_2'$ (MPa)	0.40	1.01
A	0.70	0.68
A'	0.67	0.60
$(1 - V_0/W_i)$	0.73	0.62
$(\rho_1/\rho_1' + \rho_2/\rho_2')/2$	0.44	0.30

The images show the large-scale extent of the mixing layer is growing, while the fluid within the layer is becoming more mixed and turbulent (to be quantified later). The earliest post-shock images seem to have features similar to those seen in the initial condition images, but the gradients are somewhat sharper due to the compression from the shock wave. At the PS2 time, the layer is dominated by several spikes of heavy gas (colored black) penetrating into the mixed gas (colored blue and green). Coherent vortices are noticeable at this time and the interface contours appear relatively smooth. Some chaotic behavior (jagged contours and more mixing) occurs on the left side of the layer. The slight left-right asymmetry is due to the flow in the initial condition, where the gases are injected on the left side, leading to sharper gradients on the left than on the right. Therefore, in some images the left side appears to transition to turbulence faster than the right. By the PS3 time, the smoothness that appeared along the interface is gone and many small-scale features are present. This trend continues into the PS4 time, where the mixing layer appears to be in a fully-turbulent state (i.e. increased mixing and containing a broad range of scales). Isolated regions in the PS3 images can be noticed where the layer remains relatively smooth; these no longer exist by the PS4 time, where the full layer appears engulfed by turbulence.

The similarities between the two Mach number images are remarkable given the $\sim 2\times$ difference in interface velocity. The post-shock image number (*i.e.* PS1, PS2, etc.) denotes the shock tube window used for the image and, between the two Mach numbers, represents the same post-shock travel distance ($V_o t$). Thus the post-shock travel distance appears to qualitatively capture the turbulent evolution of the mixing layer. A few differences are apparent between the two Mach numbers. The greater compression of the $M = 2.2$ flow results in a thinner mixing layer at the same window location. The composition of the layer also appears different at the last two times, where there appears to be more $\xi = 0.75$ fluid (yellow colors) in the $M = 2.2$ images.

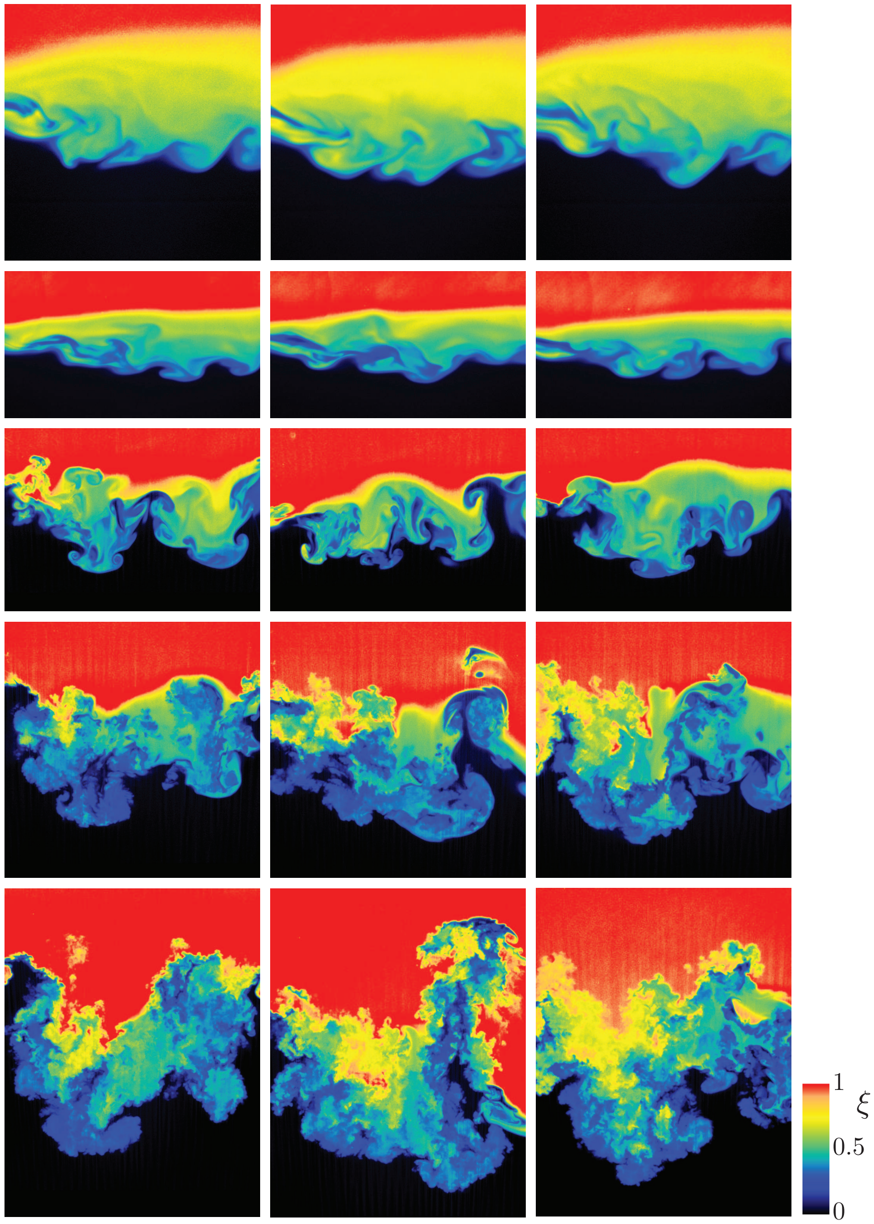

FIGURE 3. Selected images from $M = 1.6$ experimental sequence. First row: initial condition images. Second row: PS1, 0.14 ms after shock interaction. Third row: PS2, 0.88 ms after shock interaction. Fourth row: PS3, 2.16 ms after shock interaction. Fifth row: PS4, 3.84 ms after shock interaction. The width of each image is 14.0 cm.

3.2. *Mixing-Layer Thickness*

The thickness of the mixing layer, h_{5-95}, is defined as the distance between spanwise-averaged mole-fraction values of $\langle\xi\rangle = 0.05$ and 0.95, where

$$\langle\xi\rangle = \frac{1}{L_x}\int_0^{L_x} \xi \, dx \ . \tag{3.1}$$

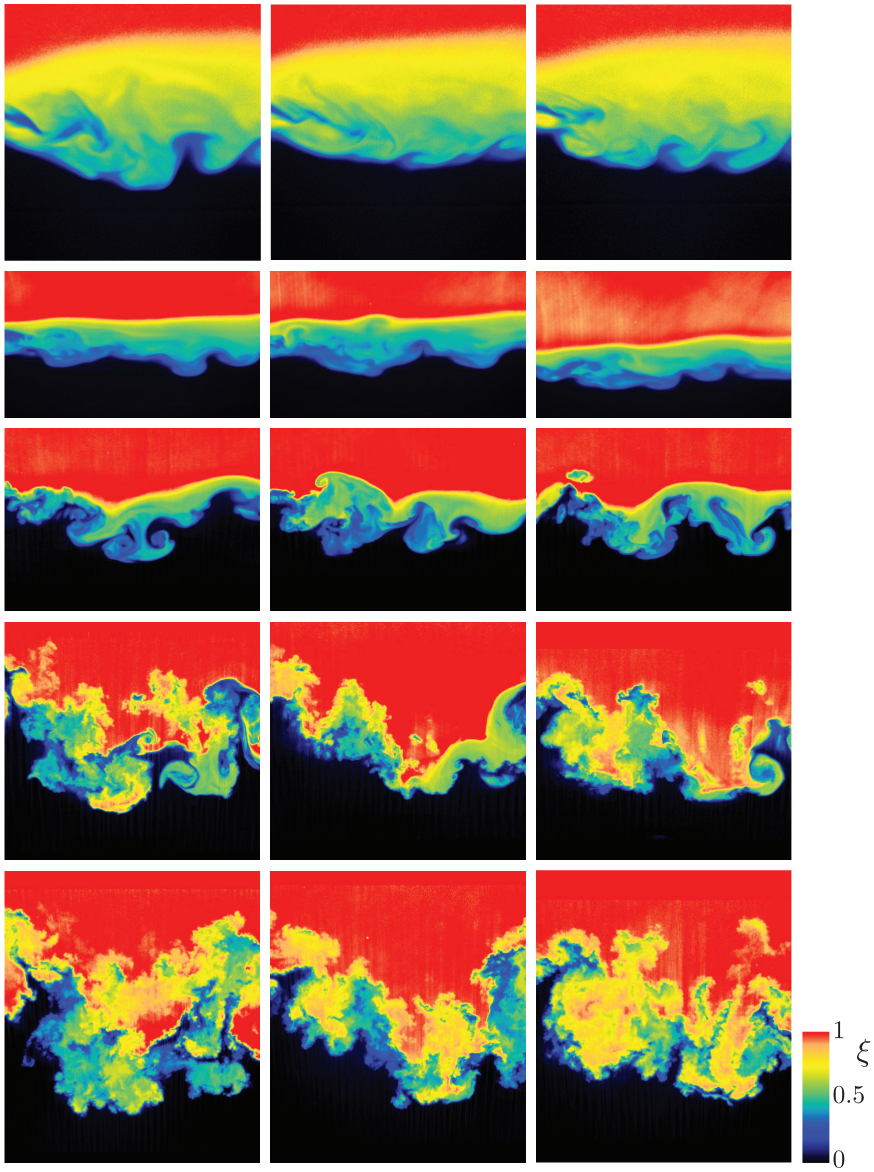

FIGURE 4. Selected images from $M = 2.2$ experimental sequence. First row: initial condition images. Second row: PS1, 0.10 ms after shock interaction. Third row: PS2, 0.44 ms after shock interaction. Fourth row: PS3, 1.12 ms after shock interaction. Fifth row: PS4, 2.05 ms after shock interaction. The width of each image is 14.0 cm.

These values are shown in dimensional form in Fig. 5(a) from all of the experimental data. Since these images show a two-dimensional slice from a three-dimensional layer, a large amount of experimental variation is expected, and the average from the experiments, shown as open circles, is of the most relevance. A weighted, least-squares regression is used to calculate the initial linear growth rate at the earlier times. The inverse of the number of images at that post-shock time is used as the weighting in the regression, ensuring that the different times contribute equally despite having different numbers of images. The growth rates from the PS1-PS2 data are $\dot{h}_{5-95} = 24.2 \pm 1.6$ m/s and 35.4 ± 7.3

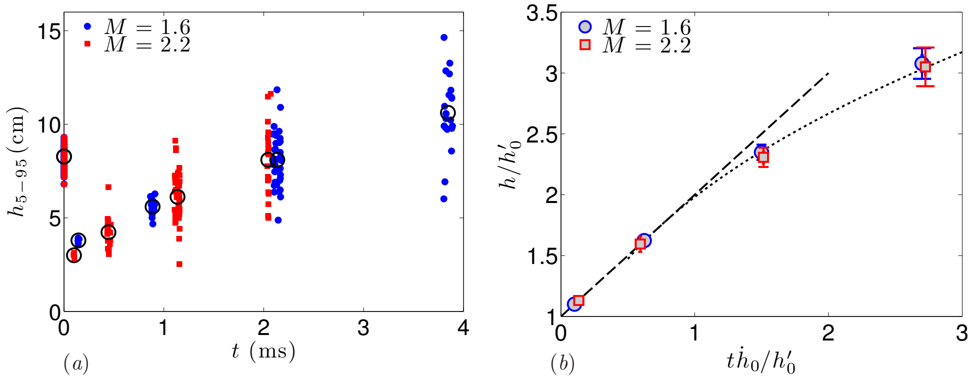

FIGURE 5. Mixing-layer thickness, h_{5-95}. (*a*) Dimensional and (*b*) non-dimensional. Error bars in (*b*) show the standard error of the mean. Curve fits in (*b*) show a linear fit (dashed) and a power-law fit (dotted).

m/s for $M = 1.6$ and $M = 2.2$, respectively. The $+/-$ value is the standard error in that measurement. The growth rates between the PS2 and PS3 data are $\dot{h}_{5-95} = 20.0 \pm 2.1$ m/s and 27.0 ± 4.1 m/s for $M = 1.6$ and $M = 2.2$, respectively, suggesting that the layer has departed from its linear-growth stage within this time frame.

The non-dimensional mixing-layer thickness is shown in Fig. 5(*b*). The thickness after shock-compression, h'_0, and the initial growth rate of the mixing layer, $\dot{h}_0$, provide excellent collapse of the two Mach number data. Also shown is the line

$$\frac{h}{h'_0} = \frac{\dot{h}_0}{h'_0}t + 1 \tag{3.2}$$

and the power law

$$\frac{h}{h'_0} = a\left(\frac{\dot{h}_0}{h'_0}t\right)^{\theta} . \tag{3.3}$$

The power law fits the last three post-shock times with values of $a = 1.98 \pm 0.01$ and $\theta = 0.43 \pm 0.01$. This value of θ is in the upper range of previously reported values ($0.25 \leqslant \theta \leqslant 0.5$) in Dimonte & Schneider (1997); Prasad *et al.* (2000); Dimonte & Schneider (2000); Jacobs *et al.* (2013).

Both h'_0 and $\dot{h}_0$ were obtained from the linear fit to the PS1 and PS2 data, but these values can be estimated *a priori* if their dependence on the initial conditions is known. The compression of the layer by the shock wave is a complex process, but a simple approximation based on the pre- to post-shock density ratios, $h'_0/h_0 \approx (\rho_1/\rho'_1 + \rho_2/\rho'_2)/2$, is within 10% of the measured values. One might expect the growth rate to behave similar to the growth rate of mixing layers after reshock, where a second shock wave interacts with the mixing layer. The interface prior to reshock in experiments like Vetter & Sturtevant (1995) and Leinov *et al.* (2009) is characterized by a thickness that contains, presumably, a broad distribution of perturbations interspersed within the layer - comparable to the initial condition in the present experiment. Reshock experiments found that the layer grows linearly after reshock and the growth rate is independent of the pre-reshock initial conditions. The mixing-layer thickness after reshock appears to fit the form of

$$h = C_M A' V_0 t + h_0 \ , \tag{3.4}$$

which was proposed by Mikaelian (1989) and fits a number of experiments (Read 1984;

Youngs 1984; Vetter & Sturtevant 1995; Leinov *et al.* 2009) with $C_M \approx 0.28 - 0.49$. Therefore the growth rate is $\dot{h} = C_M A' V_0$, which is in contrast to the growth rate of a single-mode initial condition, which is proportional to the amplitude-to-wavelength ratio. The effect of initial conditions can be investigated by looking at the ratio of initial growth rates (*i.e.* between PS1 and PS2) from the higher and lower Mach number cases, $\dot{h}_{M=2.2}/\dot{h}_{M=1.6} = 1.46 \pm 0.32$. If the growth rate were independent of initial conditions, a ratio of $(A'V_0)_{M=2.2} / (A'V_0)_{M=1.6} = 1.72$ would be expected. Alternatively, if the growth rate were also proportional to the compressed interface thickness the ratio would be 1.14. The compression due to density changes is not the only factor in the interface compression. The differences in the incident shock wave velocity, W_i, and the post-shock interface velocity will cause perturbations to decrease by a factor $(1 - V_0/W_i)$. Using this, a growth-rate ratio of 1.46 is expected, which is equal to the experimentally measured ratio. This suggests that the interface perturbations still factor into the measured initial growth rate.

3.3. *Mixing-Layer Composition*

The composition of the mixing layer is explored through probability density functions (PDFs) of mole fraction. These are shown in Fig. 6 as a function of z-location within the layer; darker colors represent an increased probability of mole fraction at a given location. Note that integrating this two-dimensional PDF over ξ will result in $\langle\xi\rangle$, which, by definition, is $\langle\xi\rangle = 0.5$ or 0.95 at $z/h_{5-95} = -0.5$ or 0.5, respectively. The uncertainty in these PDFs can be estimated from the region of uniform concentration. Due to shot noise and uncorrected features in the laser sheet, the standard deviation in this region is 0.03 for the IC and 0.04 for the post-shock images.

The first row of Fig. 6 shows that the initial condition mole fraction gradually transitions from $\xi = 1$ at the top of the mixing layer to $\xi \sim 0.4$ near the bottom. This gradual marching behavior is due to the diffusive spreading of the inlet jet, which, as seen in Fig. 2, does not contain large-scale inhomogeneities (Kelvin-Helmholtz features) aside from the very bottom of the layer. The perturbations at the bottom of the mixing layer show up as a wide region in the PDF, containing mole fractions between $\xi = 0$ and $\xi = 0.4$.

The second post-shock realization (second row) shows a PDF that has spread out over a wider extent of mole fractions; nearly all of the mixing layer contains a finite probability between $\xi = 0$ and $\xi = 0.6$. The transition between $\xi = 0.6$ and $\xi = 1$ is still confined to the top of the mixing layer as the growing perturbations that began on the bottom of the layer have not significantly influenced this region yet.

By the PS3 time, the perturbations have reached the top of the layer, entraining high mole fraction gas and distributing it throughout the mixing layer. This is evident by the $\xi = 0.6$ to $\xi = 1$ mole fraction now having a larger probability throughout most of the mixing layer. By this time, the peak in the PDF of mixed fluid ($\xi \sim 0.4$) is significantly reduced in the $M = 2.2$ experiments.

At the latest time, the transition from $\xi = 0$ to $\xi = 1$ is more gradual than earlier times, particularly in the $M = 2.2$ experiments. The peak near $\xi \sim 0.4$ still exists at the lower Mach number but is almost completely removed through mixing in the higher Mach number experiments.

The evolution of PDF(ξ) within the full mixing layer (within $0.05 < \langle\xi\rangle < 0.95$) is shown in Fig. 7 for the (*a*) $M = 1.6$ data and (*b*) $M = 2.2$ data. The PDFs show that local peak near $\xi \sim 0.4$ reduces over time and appears to mix with the lighter ($\xi = 1$) fluid. This process occurs more rapidly in the $M = 2.2$ case and results in an increase in the fluid near $\xi \sim 0.8$. This bias for mixing of the lighter fluids has been noticed elsewhere and is attributed to the greater inertia of the heavy fluid (Livescu & Ristorcelli 2008).

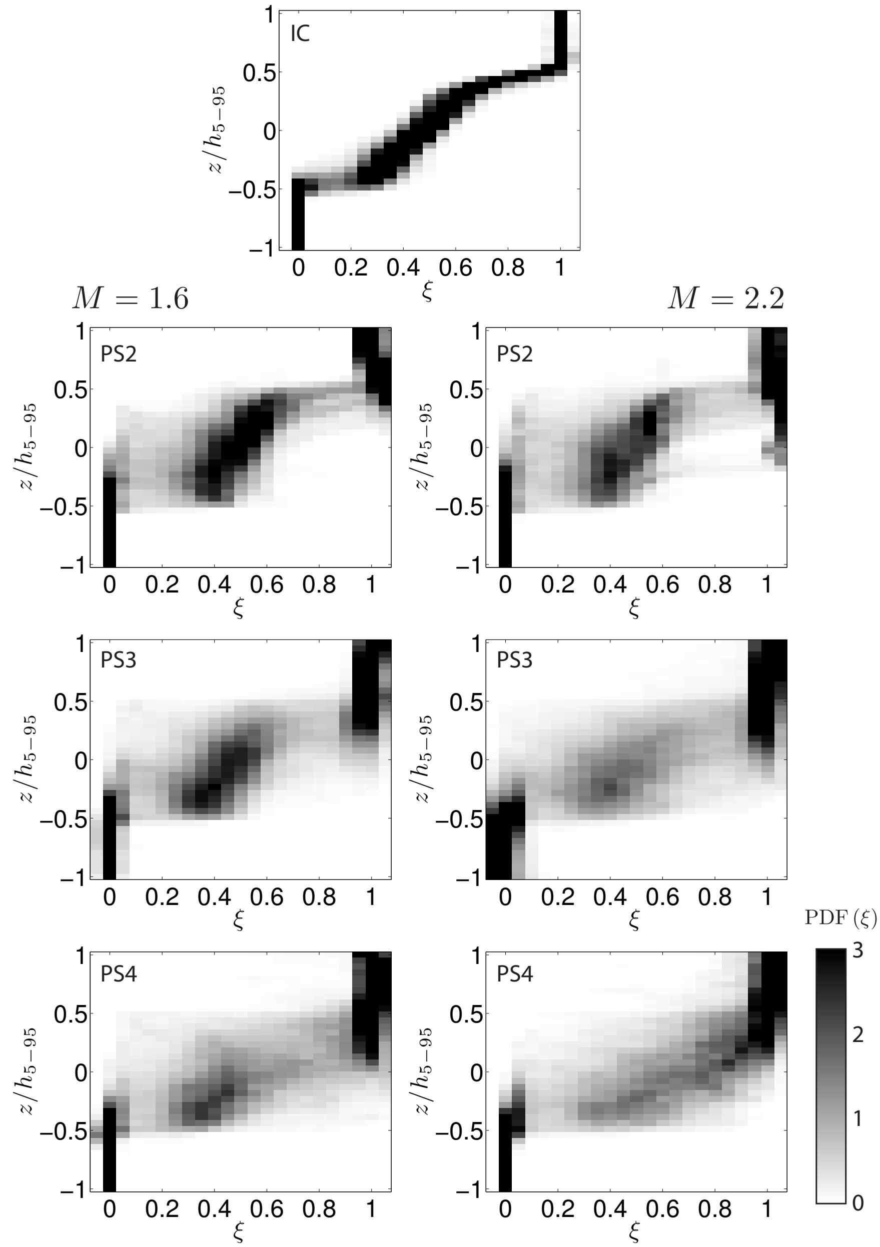

FIGURE 6. PDF of mole fraction throughout the mixing layer for the (top) IC, and PS2-PS4 from the (left) $M = 1.6$ experiments and (right) $M = 2.2$ experiments. The z-axis is normalized by the $\langle \xi \rangle = 0.05$ to 0.95 thickness, h_{5-95}.

A metric describing the state of mixing can be constructed from the ratio of the "thickness of mixed fluid" to the mixing layer thickness. Mixed fluid is defined following Cook & Dimotakis (2001) as limited by the lesser component in a stoichiometric mixture,

$$\xi_m(\xi) = 2 \min(\xi, 1 - \xi) , \tag{3.5}$$

thus an equal mixture would have $\xi_m = 1$. With this definition, the mixing layer thickness

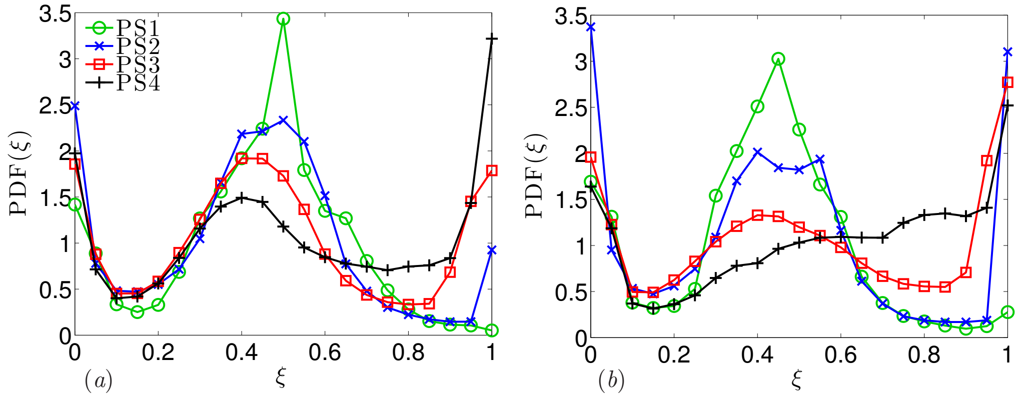

FIGURE 7. PDF of mole fraction within $0.05 < \langle\xi\rangle < 0.95$, (*a*) $M = 1.6$ and (*b*) $M = 2.2$.

is

$$h_m = \int_{-\infty}^{\infty} \xi_m\left(\langle\xi\rangle\right) \mathrm{d}z, \tag{3.6}$$

and is shown in Fig. 8(*a*) to be proportional to h_{5-95} ($h_m = 0.57h_{5-95}$). These definitions of mixing-layer thickness do not differentiate between mixed gas and unmixed but interpenetrating gas. These are compared in the following ratio:

$$\Xi = \frac{\int_{-\infty}^{\infty} \langle\xi_m\left(\xi\right)\rangle \mathrm{d}z}{\int_{-\infty}^{\infty} \xi_m\left(\langle\xi\rangle\right) \mathrm{d}z}, \tag{3.7}$$

where the denominator is the same as Eq. (3.6) and the numerator averages after converting the mole fraction field into a mixture fraction field. A fully homogenized fluid without interpenetrating perturbations will have a ratio of $\Xi = 1$, while a discontinuous interface with perturbations would have $\Xi = 0$. In these experiments this ratio, shown in Fig. 8, begins near $\Xi = 1$ for both IC and PS1, signifying that the thickness of the layer mostly comes from mixed fluid and not from perturbations. The ratio decreases at all times in the $M = 1.6$ data, and decreases before bouncing back at the latest time in the $M = 2.2$ data. Both Mach numbers reach a final value of 0.79, which is close to the asymptotic value of 0.8 reported after the onset of turbulent mixing in Rayleigh-Taylor simulations Cook *et al.* (2004).

3.4. *Density Self-Correlation*

An additional measure of fluid mixing and an important quantity for turbulence modeling is the density self-correlation,

$$b = -\left\langle \rho^* \left(\frac{1}{\rho}\right)^* \right\rangle, \tag{3.8}$$

where asterisks denote spanwise variations, i.e. $\rho^* = \rho - \langle\rho\rangle$. In the variable-density Reynolds averaged equations, b appears in the production term for the mass flux and requires modeling for closure (Besnard *et al.* 1992). One approach for closure is to make a Boussinesq approximation (Grégoire *et al.* 2005), in which case b reduces to

$$b \approx \frac{\left\langle {\rho^*}^2 \right\rangle}{\left\langle\rho\right\rangle^2}. \tag{3.9}$$

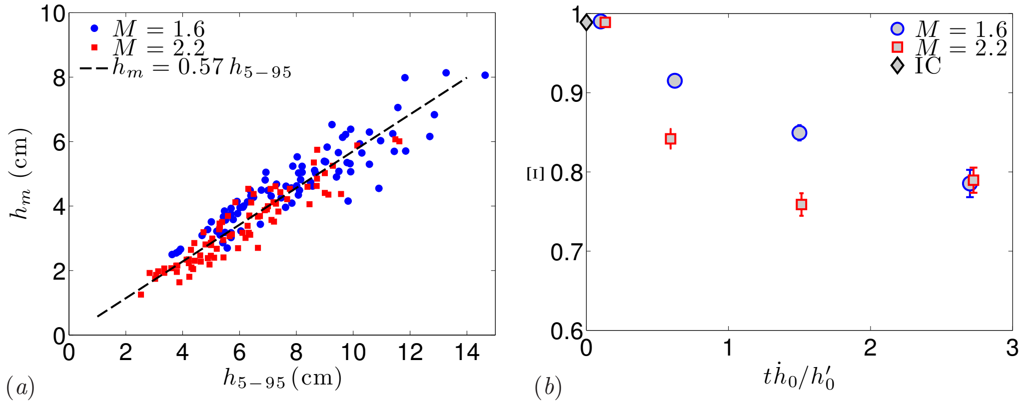

FIGURE 8. (a) Comparison of mixing layer thickness definitions, Eq. (3.6) vs h_{5-95}. (b) Ratio of the mixed fluid thickness to the mixing layer thickness, Eq. (3.7).

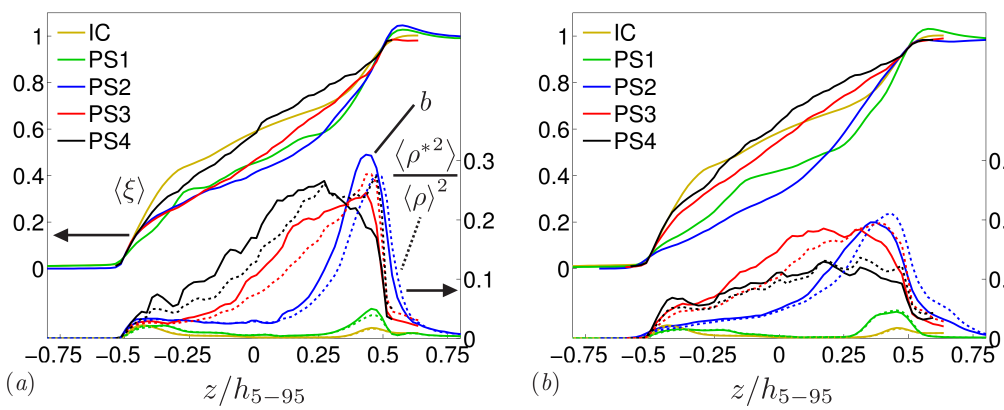

FIGURE 9. Density self-correlation, b (right axis, solid), normalized density variance, $\langle \rho^{*2} \rangle / \langle \rho \rangle^2$ (right axis, dashed), and mean mole fraction, $\langle \xi \rangle$ (left axis), (a) $M = 1.6$ and (b) $M = 2.2$.

This approximation has been found adequate at modest Atwood numbers (Livescu *et al.* 2009; Ristorcelli *et al.* 2013).

Here the density field is approximated as

$$\rho = \rho_2' + (\rho_1' - \rho_2')\xi. \tag{3.10}$$

Figure 9 shows the spanwise-averaged profile across the layer, $\langle \xi \rangle$ (left axis), b (right axis, solid), and $\langle \rho^{*2} \rangle / \langle \rho \rangle^2$ (right axis, dashed) for (a) $M = 1.6$ and (b) $M = 2.2$. The compression of the layer by the shock wave reduces $\langle \xi \rangle$ in the interior of the layer (i.e. at $z/h_{5-95} = 0$), which then increase as the layer evolves in time, but does not yet show a self-similar profile at the latest two times. The density self-correlation at the first post-shock time is similar to its value in the initial condition and both are much smaller than at later times. At PS2 a large peak in the b profile appears near the lower density edge of the layer. By the latest two times the peak is centered closer to the center of the layer but the profile is still shifted towards the lower density side. In the $M = 1.6$ case, b is still increasing through most of the layer between the latest two times, but in the $M = 2.2$ case b is decreasing by the latest time suggesting it is in a more well-mixed state. Aside from the late-time differences, the trends for the two Mach numbers are similar, but the values for the lower Mach number case are 50% larger.

The values of b measure here are a factor of 2-3 times larger than those measured in the gas-curtain RMI experiments (Balakumar *et al.* 2012) and in gas-channel RTI experiments (Banerjee *et al.* 2010). This is likely due to the presence of unmixed spikes in these experiments that protrude through the mixing layer even at late times. Despite the large range of densities, the Boussinesq approximation, $\left\langle \rho^{*2} \right\rangle / \left\langle \rho \right\rangle^2$, closely tracks the trend observed in b. As noted by Livescu *et al.* (2009) for RTI simulations, $\left\langle \rho^{*2} \right\rangle / \left\langle \rho \right\rangle^2$ tends to over-predict b on the low-density side of the layer and under-predict it on the high density side.

3.5. *Scalar Variance Spectra*

The scale distribution of the mole fraction fields is reported here using one-dimensional scalar variance energy spectra. The spectra are computed horizontally within the region $0.1 < \langle \xi \rangle < 0.7$. To reduce the influence of noise, an interlacing technique is used (Kaiser & Frank 2007) where the Fourier coefficient, $F(\xi(x))$, is multiplied by the complex conjugate of the Fourier coefficient of the adjacent row,

$$E(k_x) \approx F(\xi_j(x))F^*(\xi_{j+1}(x)). \tag{3.11}$$

Since photonic shot noise is uncorrelated from pixel to pixel, and assuming neighboring rows record a similar turbulence structure, the noise contribution tends towards zero with an average of these interlaced spectra. The spectra were found to be converged with 10-20 images.

Figure 10 shows the spanwise 1D energy spectra for the five times from the (a) $M = 1.6$ and (b) $M = 2.2$ data. The spectra from the IC and PS1 lie very close to each other, as would be expected given the very early time of PS1. Between PS1 and PS2 the magnitude of the spectrum increases, representing an increase in scalar variance. The spectra of the last three times, PS2-PS4, are very similar, which is interesting given the visual difference between the corresponding images in Figs. 3 and 4. The magnitude of the high wavenumber region is increasing through the latest time. An apparent $k^{-5/3}$ inertial range is noticeable at the latest three times. This inertial range manifests for approximately a decade in wavenumbers before an exponential dissipation region is observed. The compensated spectra are shown in 10 (c) and (d) for the $M = 1.6$ and $M = 2.2$ data, respectively and show a region between 0.7 cm^{-1} and 7 cm^{-1} that is nearly flat, although a slight negative slope appears present. A least-squares fit to the 1 cm^{-1} $< k <$ 5 cm^{-1} region finds a $E \propto k^{-1.78}$ scaling for the PS4 data at both Mach numbers.

3.6. *Turbulent Length Scales*

In this section, turbulent length scales are measured from within the mixing layer. Of primary interest are the Taylor microscale and the viscous scale, as their relationship governs turbulent mixing. From these length scales, a Reynolds number is computed and the turbulent transition is discussed.

3.6.1. *Batchelor Scale*

The inner viscous scale separates the inertial range from the dissipation range, but this can be difficult to identify in an experimental spectrum. Instead, the spectrum can be compared with a model velocity spectrum, where the relevant length scales are known (Wang *et al.* 2007; Petersen & Ghandhi 2011). Pope (2000) proposed the following model for 3D isotropic, homogeneous turbulence:

$$E_{3D}\left(k\right) = C_p \left\langle \varepsilon \right\rangle^{2/3} k^{-5/3} f_L\left(kL\right) f_\eta\left(k\eta_k\right), \tag{3.12}$$

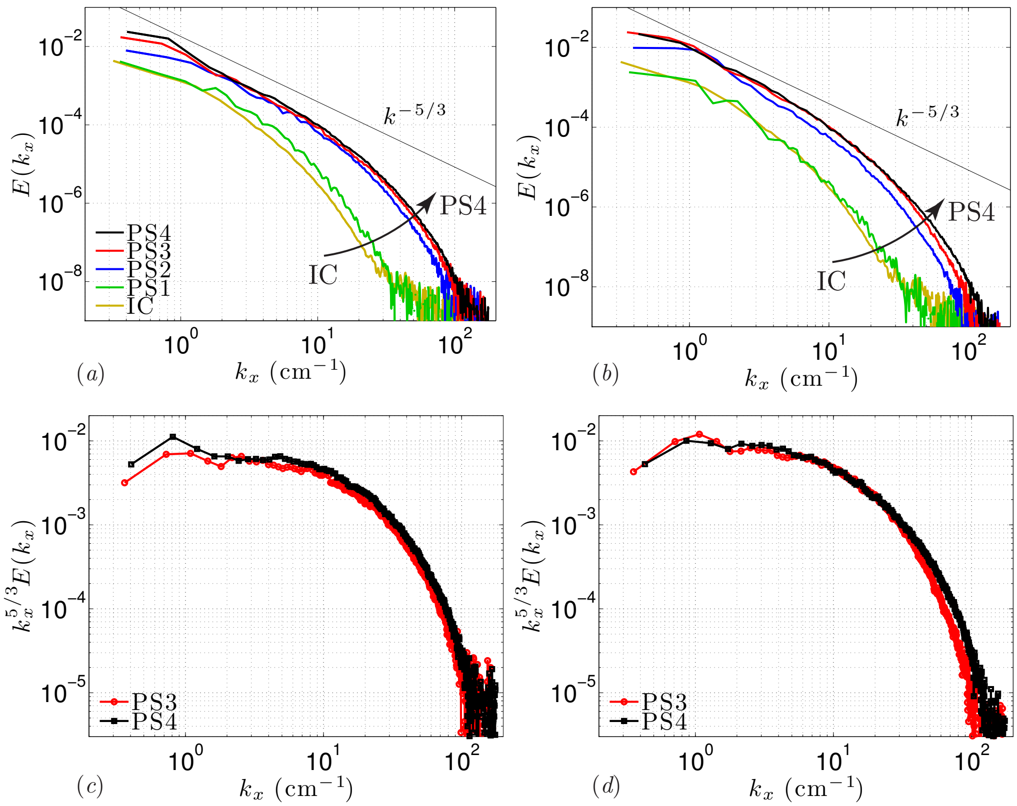

FIGURE 10. One-dimensional scalar variance energy spectra, (*a*) $M = 1.6$ and (*b*) $M = 2.2$.

with a dissipation region

$$f_\eta\left(k\eta_k\right) = \exp\left(-\beta\left(\left[\left(k\eta_k\right)^4 + c_\eta^4\right]^{1/4} - c_\eta\right)\right). \tag{3.13}$$

where η_k is the Kolmogorov length scale and β and c_η are chosen to fit experimental data. Pope found an excellent fit to a range of homogeneous, isotropic experimental data with $C_p = 1.5$, $\beta = 5.2$, and $c_\eta = 0.4$. Alternatively, the dissipation region is a simple exponential when $c_\eta = 0$, and β is constrained to $\beta = 2.1$ by requiring Eq. (3.12) to integrate to the net turbulent kinetic energy and dissipation. The one-dimensional energy spectrum is obtained through integration of the three-dimensional spectrum (Tennekes & Lumley 1972)

$$E\left(k_1\right) = \int_{k_1}^{\infty} k^{-1} E_{3D}\left(k\right) \mathrm{d}k. \tag{3.14}$$

The dissipation spectrum is related to the energy spectrum by $D(k) = 2\nu k^2 E(k)$. Using Pope's model spectrum, one finds that the dissipation spectrum reaches 2% of its peak at $k\eta_k = 1$. Therefore, measuring the 2% dissipation level allows one to infer the Kolmogorov scale. This requires a resolution of $\pi\eta_k$. If sufficient resolution or signal level is unavailable, it is possible to fit Pope's model to the resolved portion of the spectrum using η_k as a fitting parameter. In passive scalar turbulence, the smallest length scale is the Batchelor scale, λ_B, which is related to the Kolmogorov length scale by the Schmidt number ($Sc = \nu/\mathscr{D}$), $\lambda_B = \eta_k Sc^{-1/2}$, where $Sc \approx 1$ for gases.

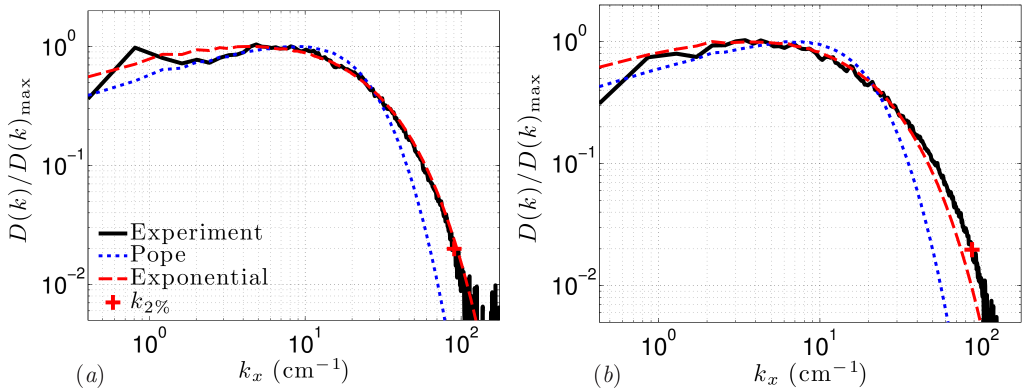

FIGURE 11. Dissipation spectrum from the latest post-shock time and model spectra, (*a*) $M = 1.6$ and (*b*) $M = 2.2$.

The dissipation spectra from the PS4 data are shown in Fig. 11. Also shown are Pope's model and the exponential version of Pope's model (*i.e.* $c_\eta = 0$ in Eq. (3.13)) with η_k adjusted to best-fit the experimental spectrum. Pope's model appears to fall off more sharply than the experimental data and a better-fit is observed at high wavenumbers by the exponential model. Pope showed that his model matched the dissipation region of grid turbulence and boundary layer data better than the other models, thus the disagreement with Pope's model also represents a disagreement with other experimental data. Also shown in the figures is the point where the dissipation spectrum reduces to 2% of its peak. The $1/k_{2\%}$ scale occurs at a wavelength of $2\pi/k_{2\%}$, requiring a resolution of $\pi/k_{2\%}$ or $\sim$350 μm, which is approximately equal to the FWHM of the line-spread function, *i.e.* approching the present experiments' resolution limits.

An additional length scale based on the Batchelor scale - the dissipation layer thickness - is computed as a comparison to the length scales computed from the dissipation spectra. The scalar dissipation rate field, $\chi = \mathscr{D}\nabla\xi \cdot \nabla\xi$ is observed consisting of sheet-like structures (Buch & Dahm 1996, 1998). These structures arise from the compressive action of the strain rate, which stretches contour lines and increases the scalar gradients, and the thickening action of diffusion. Thus an equilibrium exists where these forces balance, resulting in a scalar dissipation length scale (Su & Clemens 2003)

$$\lambda_{20\%} = \Lambda\lambda_B. \tag{3.15}$$

The proportionality constant Λ has been found to range from 2 - 14.9 (Wang *et al.* 2007; Su & Clemens 2003; Buch & Dahm 1998) and may be flow-dependent. The '20%' in Eq. (3.15) refers to a method to measure this scale: the thickness where the dissipation rate drops to 20% of its local peak.

To identify peaks in the dissipation rate field, a Canny edge detection algorithm (Canny 1986) is applied to the image of dissipation rate, which finds the local maxima of the gradients. From these peaks, the dissipation rate in directions aligned with the local gradient angle is computed through interpolation. The distance where the dissipation rate drops to 20% of the local maximum is recorded as half the $\lambda_{20\%}$ value. Points are discarded if (i) the local dissipation rate maximum is less than a given threshold, (ii) the dissipation rate does not decrease to 20% monotonically, (iii) the dissipation rate does not decrease to 20% within a certain distance, or (iv) if a value for $\lambda_{20\%}$ is not found on both sides of a local maximum. An example of this calculation is shown in Fig. 12, where the inset shows the dissipation rate and the detected $\lambda_{20\%}$ lengths.

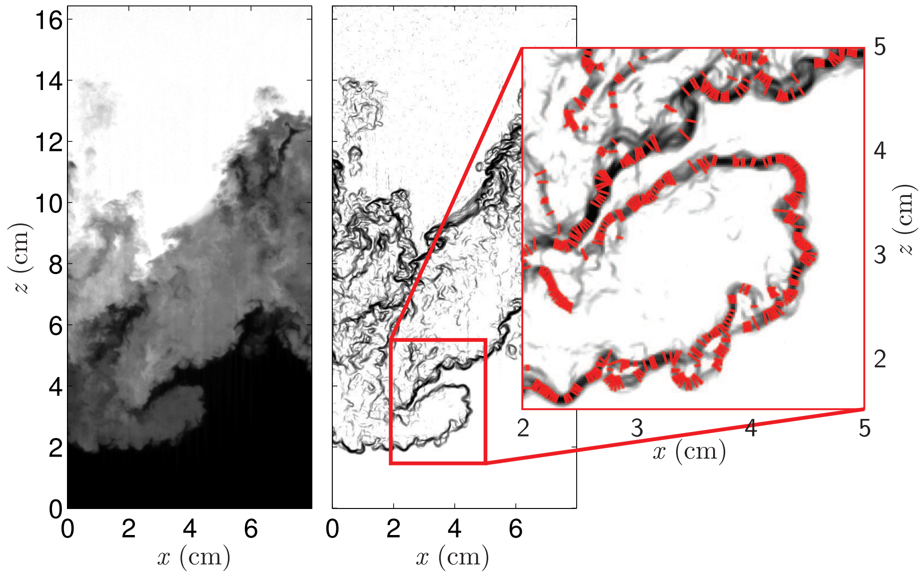

FIGURE 12. Dissipation layer thickness measurement example. Left images shows mole fraction from a $M = 1.6$ PS4 image. Middle image shows the dissipation rate. Zoomed-in image shows the 20% thickness of detected dissipation structures.

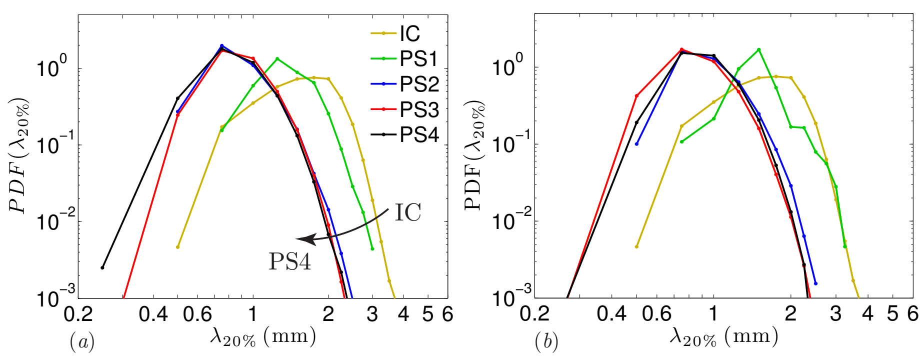

FIGURE 13. PDFs of dissipation layer thickness, (*a*) $M = 1.6$ and (*b*) $M = 2.2$.

The probability distribution function of the dissipation layer thickness is shown in Fig. 13 for the different times and Mach numbers. There is a noticeable trend of decreasing scales up until the last three post-shock times, where the PDFs all collapse to a similar curve. The peak in the late-time PDFs is at 0.75 mm, which is $\sim 5\times$ the standard deviation of the line spread function. From this resolution there is an estimated 25% error in the dissipation length scale measurements at the late post-shock times (Wang & Clemens 2004). The trend of decreasing $\lambda_{20\%}$ is notable despite the resolution limits. This scale is compared to the Batchelor scale from the dissipation spectra in Fig. 14. A proportionality constant of $\Lambda \approx 5$ scales $\lambda_{20\%}$ to λ_B at the last three post-shock times. This is within the range of previously reported values for Λ (Wang *et al.* 2007; Su & Clemens 2003; Buch & Dahm 1998). Also shown in Fig. 14 is the measured $1/k_{2\%}$ value. All of the measurements show a trend of decreasing scale for the first three times (IC, PS1, and PS2). The flattening out at the last three times may be a consequence of resolution limitations.

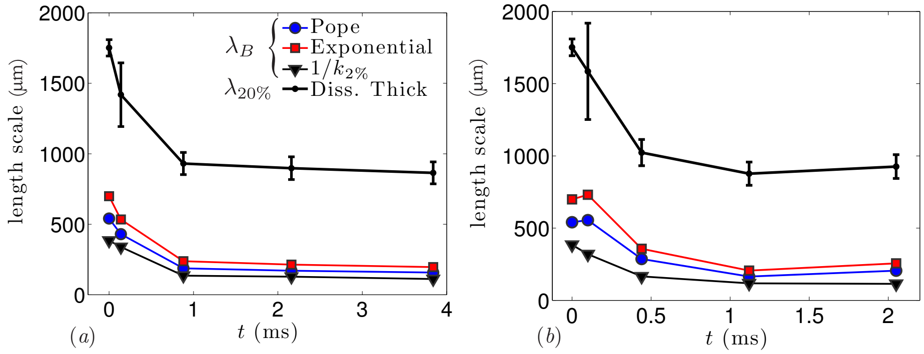

FIGURE 14. Batchelor and dissipation length scales, (*a*) $M = 1.6$ and (*b*) $M = 2.2$.

3.6.2. *Taylor Microscale*

The Taylor microscale is defined based on the curvature of the autocorrelation. The scalar variance autocorrelation,

$$R\left(r\right) = \frac{\left\langle \xi^*\left(x\right)\xi^*\left(x+r\right)\right\rangle}{\left\langle(\xi^*)^2\right\rangle} \ , \tag{3.16}$$

is symmetric, $R(-r) = R(r)$, so the first terms in the Taylor series are

$$\begin{aligned} R\left(r\right) =& 1 + \frac{1}{2}\frac{d^2R(0)}{dr^2}r^2 && (3.17)\\ =& 1 - \frac{r^2}{\lambda_T} \ , && (3.18)\end{aligned}$$

where λ_T is the Taylor microscale. This scale can be calculated directly from the curvature of the autocorrelation (Champagne *et al.* 1970; Ramaprabhu & Andrews 2004; Petersen & Ghandhi 2011),

$$\lambda_T = \left[-\frac{1}{2}\frac{d^2R(0)}{dr^2}\right]^{-1/2} \ , \tag{3.19}$$

or, equivalently, it can be calculated from the variance and the first-derivative

$$\lambda_T = \left[\frac{2\left\langle(\xi^*)^2\right\rangle}{\left\langle\left(\dfrac{\partial\xi^*}{\partial x}\right)^2\right\rangle}\right]^{1/2} \ . \tag{3.20}$$

Both methods are explored here.

From the cross-correlation theorem, the transform of the autocorrelation is equivalent to the square of the magnitude of the Fourier transform of the single variable, in this case ξ. The autocorrelation can then efficiently be computed through the inverse transform, with appropriate normalization. We take advantage of this and use the interlacing technique, Eq. (3.11), when computing the transform. This avoids some of the loss in correlation that occurs due to noise. The effect of this is small, as shown in Fig. 15(*a*), but it improves the calculation of the Taylor microscale, producing a curve that appears more parabolic near the $r = 0$ point. The full autocorrelation is shown in Fig. 15(*b*) in the horizontal and vertical directions. The horizontal autocorrelation, computed after subtracting the spanwise-averaged profile from the image, continues downward into

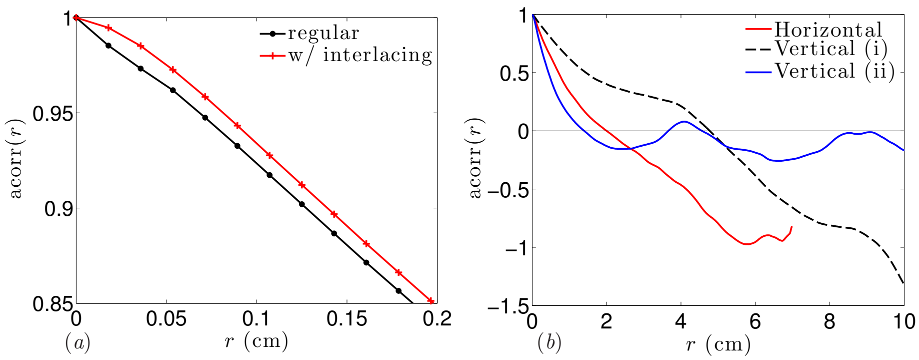

FIGURE 15. Autocorrelation examples; (a) horizontal, zoomed in to show interlacing technique and (b) the full autocorrelation in the horizontal and vertical directions (using the two subtraction methods).

the negative region due to the presence of low wavenumber structures in the layer. The vertical autocorrelation is computed in two ways: (i) subtracting the vertical average of each column before calculating the autocorrelation and (ii) subtracting both the spanwise average and the vertical average. These two methods are shown in Fig. 15(b). The vertical autocorrelation computed using the first method, subtracting the vertical average, becomes inversely (negatively) correlated over large distances. This is because the mole fraction goes from $\xi = 0$ in the bottom of the image to $\xi = 1$ in the top. This issue is mitigated by also subtracting the spanwise averaged profile from the image, which causes the autocorrelation to oscillate near zero at larger distances and is similar to the horizontal curve at small distances.

The Taylor microscale is computed from the curvature of the autocorrelation curve at $r = 0$. The chosen method is to fit a parabola to the central 7 points, *i.e.* the central $r = 0$ point, the next three points, and the equivalent three points on the negative r side of the autocorrelation. Using different numbers of points or using a second-order central difference at $r = 0$ produces proportional results but appears to have more scatter over the different experiments. As shown below, this 7-point fitting method gives similar results to a different Taylor microscale calculation in the horizontal direction.

An alternate method for calculating the Taylor microscale, used in RTI simulations (Ristorcelli & Clark 2004; Cabot & Cook 2006), is through the variance and the gradient,

$$\lambda_{T,x}^2 = \frac{2\left\langle (\xi^*)^2 \right\rangle}{\left\langle \left(\dfrac{\partial \xi^*}{\partial x} \right)^2 \right\rangle}, \quad \lambda_{T,z}^2 = \frac{2\left\langle (\xi^*)^2 \right\rangle}{\left\langle \left(\dfrac{\partial \xi^*}{\partial z} \right)^2 \right\rangle}. \tag{3.21}$$

The averaging is performed in the spanwise direction for both the horizontal and vertical directions. The results are compared in Fig. 16, where 'acorr' refers to the parabolic fit to the autocorrelation and 'var' refers to the variance/gradient method, Eq. (3.21). The two methods produce similar results in the horizontal direction, with a magnitude near 4 mm and little change in the last three post-shock times. In the vertical direction there is nearly a factor of two difference between the two methods for some of the post-shock times. Both methods start with a vertical Taylor microscale that is smaller than the horizontal scale. In the variance-based method, the scale appers to be slightly larger than the horizontal scale by the latest time, while the autocorrelation-based method produces a vertical scale that stays below the horizontal scale. The expected evolution

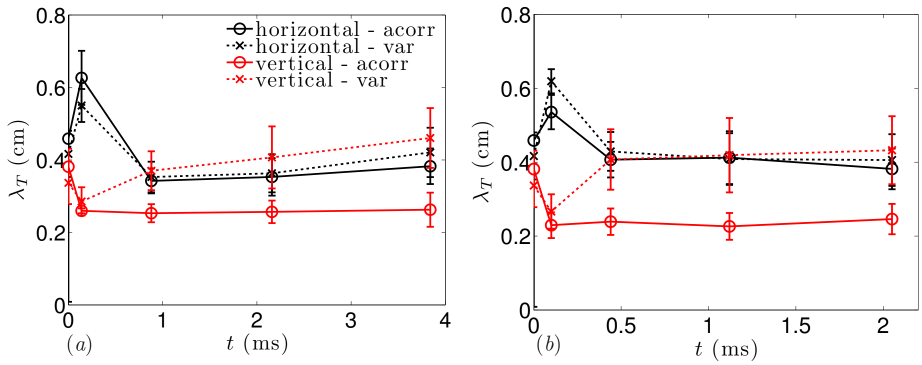

FIGURE 16. Taylor microscale, (*a*) $M = 1.6$ and (*b*) $M = 2.2$. Error bars in show the standard error of the mean.

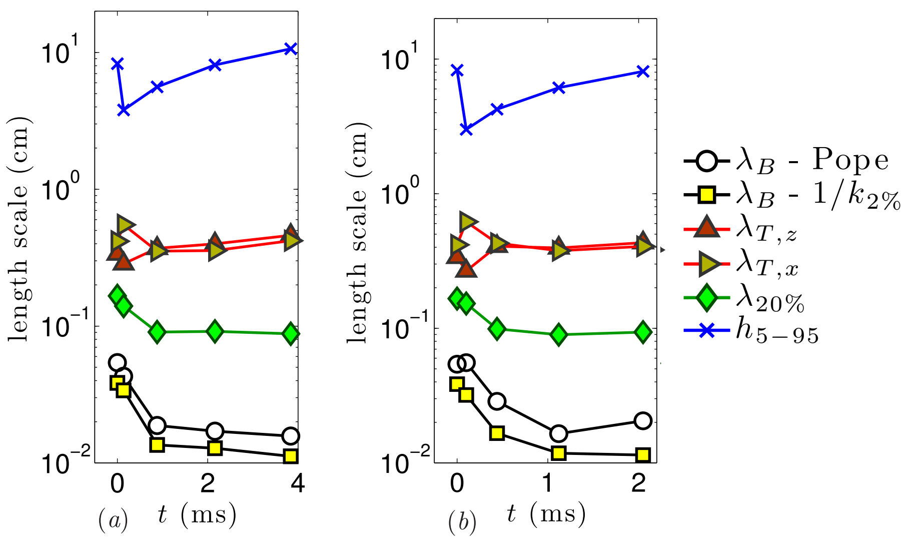

FIGURE 17. Summary of length scales, (*a*) $M = 1.6$ and (*b*) $M = 2.2$. The markers represent, from left to right, IC, PS1, PS2, PS3, and PS4.

of the layer supports the trend observed in the variance-based method. (i) The vertical scales should start smaller than the horizontal scales due to the shock-compression of the layer. (ii) In time, simulations of RTI and RMI turbulence note a persistent anisotropy with larger vertical Taylor microscales (although based on the velocity, not on scalars) (Cabot & Cook 2006; Lombardini *et al.* 2012). For these reasons, the variance-based method is used in the following discussion.

3.7. *Length Scales and Reynolds Number Discussion*

Some of the length scales previously discussed are summarized in Fig. 17. A picture of scale separation emerges from this figure, with the largest scale, h_{5-95}, getting larger, the smallest scale, $\lambda_{20\%}$ and λ_B, getting smaller, and the intermediate scale, λ_T, staying approximately the same or slightly increasing.

The Reynolds number can be measured from the Batchelor and Taylor scales. From

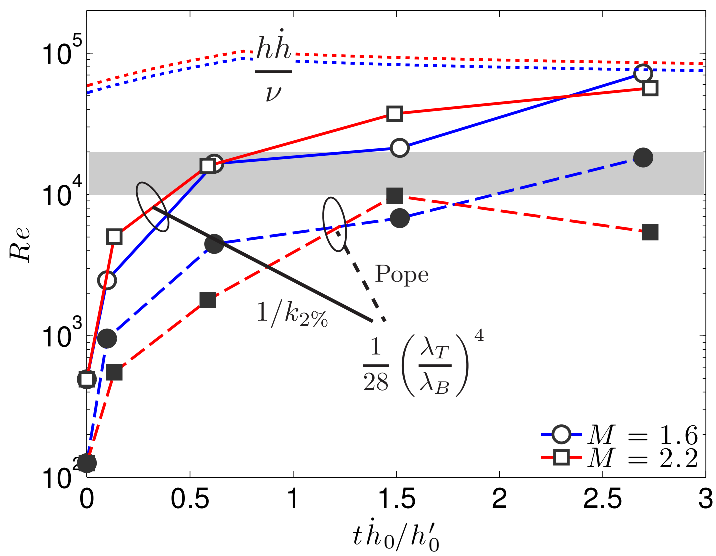

FIGURE 18. Reynolds number measurements. $M = 1.6$ in blue and $M = 2.2$ in red. Dotted lines use Eq. (3.24) and the curve-fits of Fig. 5(*b*). Solid and dashed lines use Eq. (3.23), measuring λ_B using the $1/k_{2\%}$ method and fitting to Pope's model, respectively. The markers attached to the solid and dashed lines represent, from left to right, IC, PS1, PS2, PS3, and PS4. The gray band indicates the threshold for turbulent mixing (Dimotakis 2000).

the ratio of their Reynolds number dependence (Eqs. (1.1) and (1.2)),

$$\frac{\lambda_T}{\lambda_B} = \frac{2.3\mathcal{L}Re^{-1/2}}{\mathcal{L}Re^{-3/4}}, \tag{3.22}$$

the outer-scale Reynolds number is

$$Re \simeq \frac{1}{28}\left(\frac{\lambda_T}{\lambda_B}\right)^4. \tag{3.23}$$

This is computed and shown in Fig. 18. Curves are shown for each Mach number and separate calculations of Eq. (3.23) are made using the Batchelor scale from the $1/k_{2\%}$ value and from the fit to Pope's model spectrum. The horizontal Taylor microscale, calculated using Eq. (3.21), is used. Also shown in Fig. 18 is a definition for outer-scale Reynolds number commonly used in RMI and RTI computational studies,

$$Re = \frac{h\dot{h}}{\nu}. \tag{3.24}$$

Here this equation is evaluated using the linear and power-law curve-fits from Fig. 5(*b*).

Using the length scales and Eq. (3.23), the Reynolds number grows by two orders of magnitude throughout the experiment. Using the $1/k_{2\%}$ value for the Batchelor scale, the turbulent transition (shown in as a horizontal gray band in Fig. 18) is passed near the second post-shock time. A final Reynolds number of 5.7×10^4 and 7.2×10^4 is reached at the latest time in the $M = 1.6$ and $M = 2.2$ experiments, respectively. The Reynolds number based on the Batchelor scale from fitting Pope's model spectrum is considerably smaller and does not predict that a transition to turbulent mixing would be passed. These differences highlight the error amplification that occurs in Eq. (3.23) due to the power of 4 involved. But both calculations exhibit the same trend, with a Reynolds number

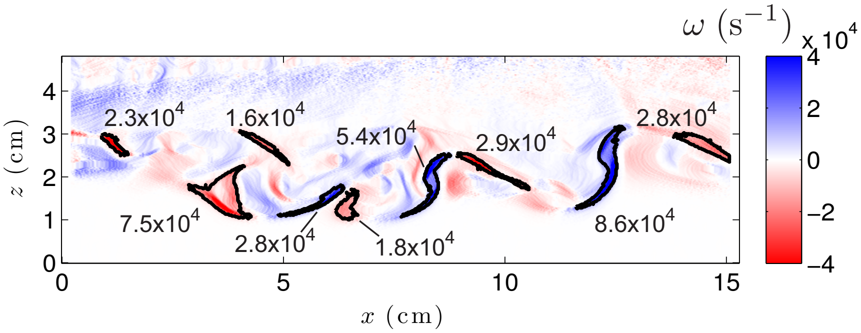

FIGURE 19. The estimated initial vorticity field from a $M = 1.6$ experiment. The vortex Reynolds number of several vorticies are noted.

increasing in time due to increased scale separation as the mixing layer develops. The Reynolds number using the $1/k_{2\%}$ value quantitatively agrees with the conclusions of the mole fraction PDFs and the spectra, where a transition to turbulence appeared to occur near the second post-shock time. The Reynolds number based on $h\dot{h}/\nu$ is much larger than the other estimates and does not capture the transition that occurs during the experiment. This is not expected to be an accurate measure for Re, at least in the development stage of the current experiments. A thicker initial value of h will increase the $h\dot{h}/\nu$ measurement without adding additional momentum to the layer.

A final estimate of the Reynolds number can be made using a model of vorticity deposition on the interface and compared to the values in Fig. 18. Assuming an impulsive acceleration to a velocity V_0, the out-of-plane vorticity deposition on the interface is (Weber *et al.* 2013)

$$\omega_y \approx -\frac{V_0}{\rho}\frac{\partial \rho}{\partial x}, \tag{3.25}$$

where ρ is the compressed, post-shock density field. This vorticity model is applied to a PS1, $M = 1.6$ image, which is taken immediately after shock compression where it is expected that little amplitude growth has occurred. Figure 19 shows the deposited vorticity approximation, using Eqs. (3.25) and (3.10).

From Fig. 19, the initial post-shock layer appears to be composed of coherent rings and tubes of vorticity. A vortex ring in isolation becomes turbulent and breaks apart when its vortex Reynolds number is larger than 2.5×10^4 (Glezer 1988). Vortex tubes in this layer will experience similar dynamics, complicated by interactions with neighboring vorticies, thus it is appropriate to measure vortex Reynolds number within this layer. The vortex Reynolds number is defined as

$$Re_\Gamma = \frac{\Gamma}{\nu} \tag{3.26}$$

where Γ is the circulation of the vortex core.

Vortex cores can be identified in the vorticity field by isolating regions where the vorticity magnitude is above a certain threshold and integrating the vorticity, $\Gamma = \int \omega \, da$, within this region. Several vortex cores are outlined in Fig. 19 and their corresponding vortex Reynolds number are noted. This method finds a number of vorticity regions that are above the turbulent threshold, with a maximum of $Re_\Gamma = 8.6 \times 10^4$. This value is similar to that measured using the Taylor/Batchelor scale ratio. Thus the Reynolds number assembled from the measured turbulence scales appears to support the other analyses of the PLIF images, showing a turbulent transition by the third post-shock time.

4. Conclusions

The turbulent mixing that results from the Richtmyer-Meshkov instability was studied using a unique shear-layer initial condition and quantitative PLIF imaging. After acceleration by a $M = 1.6$ or $M = 2.2$ shock wave, the mixing layer is initially dominated by the growth of large-scale spikes and bubbles, but these structures eventually break apart into smaller scales, leading to molecular mixing and scale separation that is indicative of a turbulent transition. The two Mach numbers seem to evolve similarly when compared at the same interface travel distance.

The mole fraction PDFs and spectra provide evidence for turbulent mixing at late times. The PDFs show three peaks, two representing the unmixed fluids and one representing the mixed fluid that is present in the initial condition. The central peak in the PDF reduces in time and disappears by the latest time at the larger Mach number. This intermediate fluid and the light fluid are observed mixing to produce $\xi \sim 0.8$ fluid. The scalar variance energy spectra appear to be fully developed by the last two times and exhibit an inertial range close to $k^{-5/3}$.

Several length scales from within the mixing layer are measured and provide a clear picture of the scale separation that causes the turbulent transition. The large scale extent of the mixing layer is found growing linearly early on and then as $t^{0.43}$ by the end of the experiment. The smallest length scale, the Batchelor scale, reduces from its initial condition value and reaches 100 - 250 µm, depending on the technique used. The intermediate scale, the Taylor microscale, shows early-time anisotropy caused by the shock wave and then stays near 4 mm for the latter three post-shock times.

The ratio of the Taylor to Batchelor scale is used to compute the Reynolds number. This Reynolds number is growing in time and crosses the turbulent transition threshold near the second post-shock time, eventually reaching $6 - 7 \times 10^4$. This result is dependent on the method used for measuring the Batchelor scale, but the $1/k_{2\%}$ appears to support the results of the PDF and spectral analysis. This Reynolds number is similar to the vortex Reynolds number through an estimate of the initial baroclinic vorticity.

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